Forking Rooms, Fractal Cities

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Filmic Experience of Non-3D Space Represented with Point Cloud

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1 A film still from *Fractal Cities* featuring a scanned and folded Tong lau building. A link to the film: https://vimeo.com/816010868

ABSTRACT

Recent explorations at the intersection of mathematics, computer graphics, and art present digital environments that challenge traditional flat, 3D space. They refresh the common understanding of the virtual world as a copy of reality, and suggest a fluid space reminiscent of the early digital age's dreams. In this regard, this study investigates the visualization of non-3D spatial experiences using point cloud, drawing on mathematical concepts, such as non-Euclidean, fractal, and hyperdimensional geometries to envision a digital space with a transformable structure. This is demonstrated through two films, Forking Rooms and Fractal Cities.

The paper introduces the technical and architectural methods employed in the experiments. The project manipulates point clouds using procedural modeling with the Blender geometry node, constructing spaces that can unfold, morph, and interconnect in relation to time or the spectator's location. It further proposes three spatial prototypes - portal, fractal, and tesseract - each exploring a specific topological condition that challenges a fundamental architectural concept: boundary, scale, and directionality. These prototypes serve as the vocabulary for storytelling and space-making in the production of the films.

This work serves as an examination of the medium, expanding the capabilities of point cloud, and identifying potential design opportunities in the creation of virtual environments. By outlining a working method for designers, and offering examples of its spatial and aesthetic qualities through film, the project discusses the possibility of designing non-3D space, and imagines a virtual world with a dynamic spatial structure.

INTRODUCTION

Medium Specificity of Point Cloud

Three-dimensional scanning has empowered designers across various fields, enabling them to create digital representations of reality with ever-increasing fidelity and convenience. In addition to its growing speed and precision in measurements, this technology also demonstrates its alluring affect in the making of hyperrealistic digital environments, where photo-scanned assets come together to form a convincing world that could hardly be distinguished from reality.

The narrowing gap between the imagery of the digital and the real, calls for a critical examination of the tool's unique aesthetic and spatial implications different from reality, as the room in between facilitates the medium to function as a generator of alternative imaginations toward the world. In his analysis of photogrammetry, Michael Young writes, "…no mediation has a privileged access to the real. Each produces its own traits, which come to define its qualities" (Young 2022). These traits may as well be concluded as the medium specificity (Greenberg 2018), which is grounded in the technical condition of the medium, and is aestheticized through the interpretations of designers. This study contributes to the ongoing exploration of the inherent nature of point cloud by analyzing its constraints and expanding its capacities in the production of two films and a series of prototypes (Figure 1).

Fluid Digital Space

Decades after Deleuze and Guattari's theory was introduced to the discipline of architecture, we continue to see the emergence of concrete monuments from virtual horizons. In a metaverse space named *Liberland*, designed by Zaha Hadid Architects, the virtual city exhibits a collection of curvilinear monuments with ambiguous materiality, whose fluidity remains rhetorical and formal. In most metaverses, the land and architecture remain static to facilitate quantification and monetization. Yet, as Deleuze and Guattari write in search of a mathematical definition of smooth space, which should "… [be] constituted by the construction of a line with a fractional number of dimensions greater than one, or a surface with a fractional number of dimensions greater than two (Deleuze and Guattari 1987)", can we envision a virtual world that is intrinsically non-hierarchical (or non-striated) and in flux? Can spatial topology and dimension be the scope of action in the design of digital spaces?

Advancements in computer graphics and creative practices in the entertainment industry enable the exploration of spaces that are not flat (zero spatial curvature), three-dimensional, but rather non-Euclidean, fractal, or hyperdimensional. While these concepts may be unattainable in reality, they have inspired visual fantasies and novel virtual experiences, both as direct applications and through cultural dissemination. The video game *Superlinminal* (Pillow Castle 2019), for instance, immerses players in a world with distorted scale and distance, requiring them to navigate puzzles utilizing these uncanny physical laws. Such imaginative spaces in popular culture suggest the potential for subverting static 3D structures in virtual environments, thus challenging fundamental spatial concepts like enclosure, scale, and direction. This research investigates the potential of fluid digital environments in space-making and storytelling, and represents these experiences through film.

BACKGROUND

3D Scanning in Architecture and Film

This brief overview primarily focuses on the work regarding 3D scanning as a means of representation, while excluding projects that involve capturing geometries for fabrication purposes.

Around the acquisition of scanned data, projects and writings investigate the characteristics and politics of Lidar scanning and photogrammetry. Fernández-Abascal and Grau discuss how the vision of driverless vehicles would disrupt the street space and imply a new social contract, and visualize such machine vision through point cloud renderings (Fernández-Abascal and Grau 2019). With a more speculative approach, the film *Where the City Can't See* (Young and Maughan 2016) directed by Liam Young and Tim Maughan imagines a city under surveillance, the shadow of which evades the scrutiny, and accommodates ungoverned activities. Shifting to photogrammetry and an aesthetic perspective, Michael Young exposes the distortion that occurs at the fringes of the captures, resulting in a textured landscape containing information of various resolutions (Young 2021).

Other projects foreground the materiality of point clouds or 3D scans. In *Baroque Topologies*, Andrew Saunders presents exterior views of interior Lidar scans, portraying the spatial volumes as solid bodies with intricate textures (Saunders 2018). Conversely, Artist Li Weiyi critiques the realism of 3D scans. In *The Hands of an Apostle*, she sections photo-scanned hands and reveals the void within encapsulated by a thin mesh. Expanding the definition of its materiality, *Cloud ~ing* by Alam and Profeta explores various interpretations of point cloud, representing it as granular 3D prints, mashed pixels in drawings, and porcelain or metallic spheres in videos.

Building on the precedents, this project takes a slightly different approach toward point cloud, neither critiquing the inherent biases in its generation nor examining its material associations. Instead, it treats point cloud as millions of points with unambiguous spatial coordinates that can be precisely choreographed in geometric transformations, and pushes its representational limits through animation.

Non-Euclidean, Fractal, Hyperdimensional

It has always been technically feasible to construct digital spaces beyond three dimensions, as computers do not inherently differentiate between 3D and higher dimensions, while they are ultimately rendered as 2D images. It is the visualization of non-3D space that could turn it from invisible data to a tangible experience. With the increasing power of GPU computing, and innovative rendering techniques, fractals and non-Euclidean spaces have gained popularity on screens over the past decade. The paper by Hart, Sandin, and Kauffman (1989) first proposed the method of Distance Estimation to render fractal geometries. Since around 2010, the visualization of fractal objects – the quintessential one being the Mandelbulb – became a noticeable topic in the computer graphics community, with major contributions from platforms like *Shadertoy* (Quilez and Jeremias 2013) and the blog *Syntopia* (Christensen 2008). The cumulative innovations have brought these exotic mathematical figures to a broader range of users through packaged add-ons, demos, and even video games such as *Marble Marcher* (CodeParade 2019). Video art projects represented by the work of Julius Horsthuis produce mesmerizing effects through infinite zooms into ever-developing fractal landscapes (Horsthuis 2021).

In addition to fractals, mathematicians and computer scientists have been exploring immersive and interactive experiences in non-Euclidean spaces. Past research works have explored the mathematics of visualizing the experience in hyperbolic spaces (Hart et al. 2017a, 2017b), the technical methods for real-time ray tracing rendering (Velho, de Silva, and Novello 2020), and bodily experience and interaction in non-Euclidean games (Weeks 2020). Released in 2022, the video game *Hyperbolica* presents spaces with curved horizons or reverse perspectives, and incorporates puzzles that arise from these unconventional characteristics (CodeParade 2022).

A notable architectural project addressing hyperdimensional spaces is by Canizares and Delgado from Office CA. They published two virtual spaces as non-fungible tokens, *Space Oddity – Inversion* and *Inside-Out House*, in which ordinary houses undergo 4-dimensional rotation. The interior of the houses folds and unfolds in loop animations, proposing alternative spatial prototypes achievable only in virtual reality.

Previous research and projects have successfully visualized the abstract mathematical figures, and inventively explored interactive experiences across various mediums. However, as most of them built the environment and the render engine from scratch, they remain inaccessible for many designers. This research searches for working methods with tools at hand to experiment with similar spatial concepts, and specifically situates the investigation in the discourse of architecture.

Regarding terminology, this paper refers to these concepts collectively as non-3D for simplicity, although this is not rigorous, since, for example, a non-Euclidean space can still be 3D, except that its curvature is not zero. Admittedly, non-3D speaks to common spatial experience, rather than serving as a strict mathematical definition.

METHODS

Non-3D Experience in 3D Software Environments

Although theoretically achievable, any non-Euclidean, fractal, or hyperdimensional space, conflicts with current 3D modeling software. To avoid the arduous technical task of building a non-3D engine, the first task of this research is to bypass the built-in three-dimensionality, while using existing software. The solution is to create 3D objects that change at every frame or in relation to the location of the spectator. For instance, to represent a hyperdimensional object, we could build a 3D object that changes its shape, which can be regarded as moving it along the fourth axis. Each model, at a certain frame, thus becomes a 3D section of the hyperdimensional object, whose continuous development is equivalent to a hyperdimensional experience. For a non-Euclidean space, the model at each frame is the current distorted space relative to the vantage point, and for a fractal object, the model grows more details as the view zooms in. In conclusion, non-3D objects are realized mathematically, with the model in the software environment acting as their instantaneous 3D section.

Film serves as another method to circumvent the technical difficulty of creating a real-time, non-3D experience. Given the current hardware capability, it is challenging to achieve the intended transformation of a high-resolution model as heavy as a point cloud without noticeable lag between frames. However, the objective of the project is not to provide a seamless virtual experience, but to visualize such a possibility and stimulate the discourse around it, for which film is used as a placeholder until real-time

2 Photogrammetry input and outcome. 2

3 Point cloud display in Rhino as the view zooms in. 3

4 Various material properties of point cloud in Blender. 4

experiences are technically feasible. In addition, in any individual clip of the films, the spatial transformation is not achieved through compositing, but by distorting the point cloud using camera position and time as input variables. In other words, they can become VR environments with a moveable camera if hardware allows for real-time rendering.

The spatial transformation makes point cloud an ideal medium. This operation would have been significantly harder with mesh, whose complex topology can easily lead to errors during transformation. Without face or edge structure, the point clouds used in the project only contain location and color data, reducing the work to programing how each set of x, y, and z coordinates would translate in the folding of space. In the case of a mesh model, in contrast, the transformation of the vertex-edge structure and face normal directions increases the difficulty of calculation and coding.

Project Context

In addition to the technical investigation, the project is also a reflection of the living spaces in Hong Kong. The selection and documentation of existing spaces follow two threads: interior and exterior. In the context of the Covid-19 pandemic, the team surveyed a series of quarantine rooms during lockdown, where individuals worked, rested, underwent testing, and recovered. From a typological standpoint, the exterior scans primarily focus on the various forms of collective living, including tower, courtyard, low-rise, and temporary living spaces in walkway tunnels. These interior and exterior spaces are woven together and narrated in two short films, titled *Forking Rooms* and *Fractal Cities*.

Tools

The team used COLMAP (Schönberger 2022) for photogrammetry to document the existing spaces, offering two advantages over Lidar scanning. First, in documenting interior spaces, the team could not physically visit each quarantine room with equipment; instead, they provided a video shooting guide to acquire suitable data for photogrammetry. Second, for scanning outdoor cityscapes, photogrammetry proved to be an appropriate method for capturing distant buildings (Figure 2).

The choice of the tool has been evolving over the 1.5-year development of the research, starting with Rhinoceros 3D (Robert McNeel & Associates 2020) and Grasshopper (David Rutten 2020), and ending with Blender (Blender Foundation 2022). The point cloud operation in Grasshopper was achieved with the Tarsier plug-in (Newnham and Jahn 2019). However, two limitations emerged with Grasshopper, the first one being the computation speed, as it primarily relies on the CPU's processing power. Second, point cloud in Rhino3D cannot be rendered in any compatible render engine other than its own render mode, in which each point appears as its original RGB, but stays unaffected by lighting, and retains the same size regardless of its distance from the viewer. This render attribute gives point cloud an ephemeral materiality for its fading translucency when zoomed in (Figure 3). The gaps between sampling points indicate the resolution limits of the medium, while also contributing to its aesthetic characteristics. However, this conflicts with the project's goal of visualizing convincing and tangible spatial transformations, which would benefit from volumetric representation of points.

The updated functions in Blender, with additional help from an add-on by Prostka (2022), allowed for new rendering possibilities. The point cloud data can be accessed and programmed using the built-in geometry node function, and keyframed in the timeline. By default, each point is rendered as a solid sphere, and it can be any other instance, whose material quality can be adjusted in the shader editor (Figure 4). These functions laid the foundation of a cinematographic exploration, as targeted in this project.

Prototypes

Technically, any 3D object that changes its shape over time can be regarded as a 4D object. However, this understanding generalizes the definition and obscures the unique experience that a non-3D environment can offer. The research instead selected several prototypes that are both typical mathematical cases and exemplars in popular culture, which are believed to embody both technical and aesthetic particularities of non-3D space.

(1) Portal

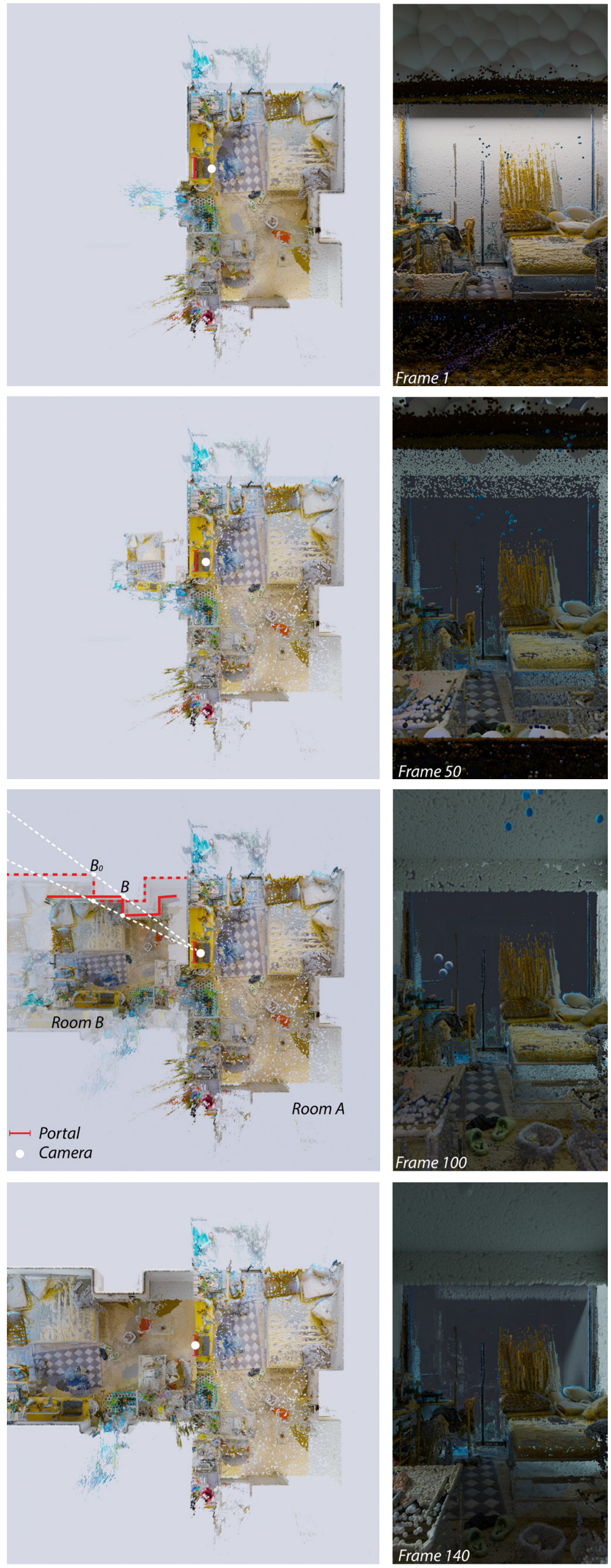
The research first attempted to build a portal, not by teleporting the camera, but by morphing the space according to the camera's location. In a demonstrative case, there are two rooms, A and B, connected through a portal (Figure 5). Initially, the camera is in Room A, from which Room B appears as a flattened image contained within a painting frame. As the camera gets closer to the portal, Room B inflates. Finally, when the camera passes the plane of the portal, Room B scales back to its original size, and the camera continues its journey inside the next room. During this process, from the camera's point of view, Room B appears the same in perspective despite its enlarging depth. To achieve this, every single point composing B should only move along the line between the camera and its original position in the undistorted point cloud B_0.

In another demo, the start and the end of the journey are identical, producing an even more uncanny experience of recursively entering the same space (Figure 6).

(2) Fractal

The premise of modeling a fractal object is that it can only be realized within a certain resolution. Despite this, we consider these objects to be "real" fractals because they have the potential to reach infinity if computational power and memory allow it. This paradoxical relationship between the conceptual infinity and finite resolution is inherent in all fractal representations, as they can be rendered, at best as accurate approximations with a tolerance smaller than one pixel on the screen.

As the definition of fractal comprises endless mathematical constructions, this research also tested different paths toward the infinitesimal. One of the initial attempts was to reference the arguably most iconic fractal object, the Mandelbrot set. A scanned apple was reflected and refracted, decreasing in size as it approaches the convoluted boundary of the figure. Other tests include a reversed stereographic projection of an infinite apple grid (Figure 7). However, these fractal objects were too expensive to compute for an animation. The “fractal” object used in the film was achieved by mirroring and scaling. To simplify, the diagrams describe the transformation in one single dimension. Repetitive stripes in the pattern of A-B-A-B slice the space. Whatever falls in A remains in its original orientation, whereas being mirrored in B. As B grows from zero and approaches A, the object is folded more densely until



5 Top view and perspective view of the journey through the portal.

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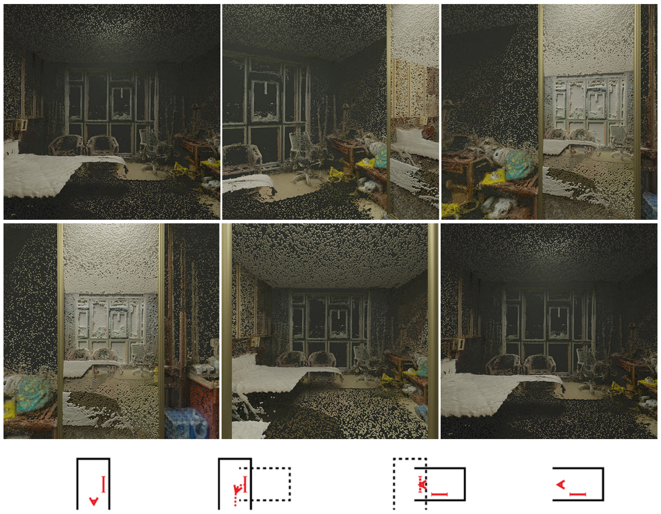

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6 Perspective view and diagrams of the tour entering the same room.

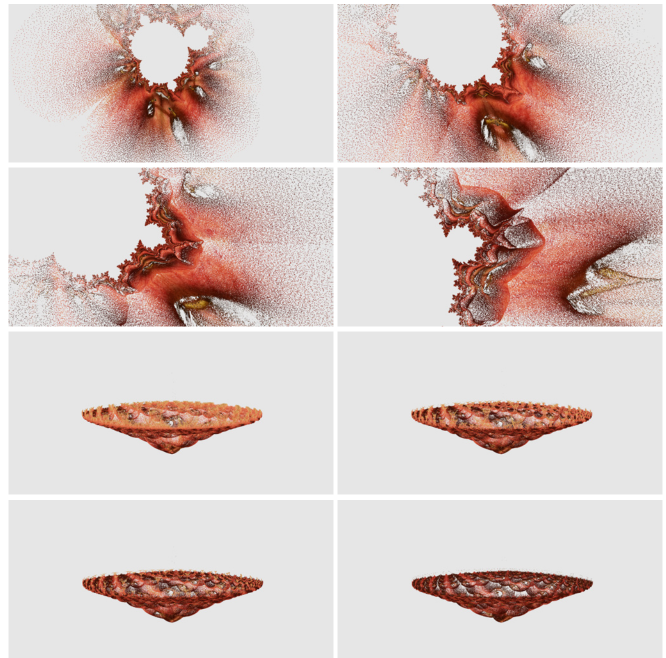
each block becomes infinitesimal (Figure 8). When the same transformation is applied in all three axes, it generates a peculiar condition where the details of the object shrink and duplicate while the overall size remains the same. However, the paper acknowledges that in the final film, the object has not been densified to infinity (within the screen resolution), but merely alludes to a true fractal.

(3) Tesseract

To be precise, the tesseract is visualized in 3D as its "perspective," and so is the image commonly presented to the public. Similar to the perspective projection of a 3D object on a 2D canvas, the 4D cube is also extruded to the vantage point, whose intersection with the 3D "canvas" becomes its perspective. Each of its cells, which is a cube, is foreshortened into a pyramidal frustum. As the vantage point orbits around it, the tesseract revolves in 3D, and the frustums are flipped inside out.

The same point cloud is placed on different cubic cells of the tesseract. In both the demo featuring a scanned Buddhist altar (Figure 9) and the final film *Fractal Cities* (Figure 10), only four out of the eight cells contained the point cloud to prevent excessive occlusion between them, thereby maintaining the legibility of the transformation. Additionally, the point cloud is mirrored in two directions on each cell, ensuring continuity with the adjacent cell.

The three prototypes above suggest three antitheses to three-dimensionality – non-Euclidean, fractal, and hyperdimensional. Furthermore, they each challenge a preconception in digital space that has been naturally inherited from the physical world. The portal redefines the concepts of in and out or the idea of *boundary*, as the 3D



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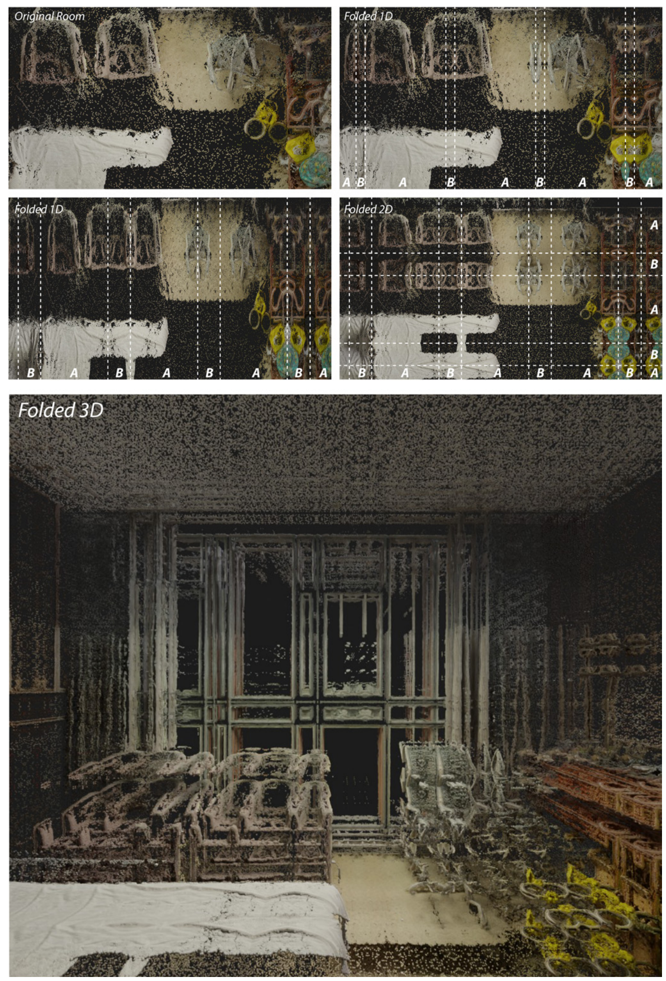

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7 Early fractal tests with a scanned apple.

8 Mirroring a scanned room in 1D, 2D, and 3D.

9 A scanned Buddhist altar revolving with a tesseract.

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10 The city unfolds as the tesseract revolves in *Fractal Cities*.

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space may not be fully enveloped by a bubble, but could be teleconnected inward. The fractal interrogates *scale*, as large and small are relative and interchangeable measures. The tesseract disrupts *directionality*, as a person would return to the same room after walking through three others in a row. By disturbing in/out, big/small, and front/back, which are fundamental metrics that have regulated the perception and construction of architecture, the project offers new possibilities for what the digital space could be.

Films

In pursuit of unique spatial and aesthetic characteristics, the research employs the spatial vocabulary in the creation of two films, one illustrating an interior experience and the other generating exterior building forms. The first addresses experience, which is diachronic, and the second emphasizes form, which is synchronic. In *Forking Rooms*, whose title and narrative pay tribute to Borges's novel, *The Garden of Forking Paths* (Borges 1962), showcases a first-person journey of entering enclosed domestic rooms through portals: screens, mirrors, and windows (Figure 11). Traversing a series of scanned bedrooms, the protagonist hidden behind the picture plane steps into the same room three times, each time finding the room slightly stranger, where the screens multiply and so do the worlds connected behind them.

In *Fractal Cities*, the scanned spaces in Hong Kong are reimagined as a malleable landscape, constantly folding, developing, morphing, and interconnecting (Figure 12). Beyond a truthful survey and documentary, the found places in the city are dissected and orchestrated in space and time to also metaphorically represent the unsettling urban experience.

11 The journey goes as Room A1-A2-B-C-A3, in which Room A has been entered three times. A link to *Forking Rooms*: https://vimeo.com/735693189

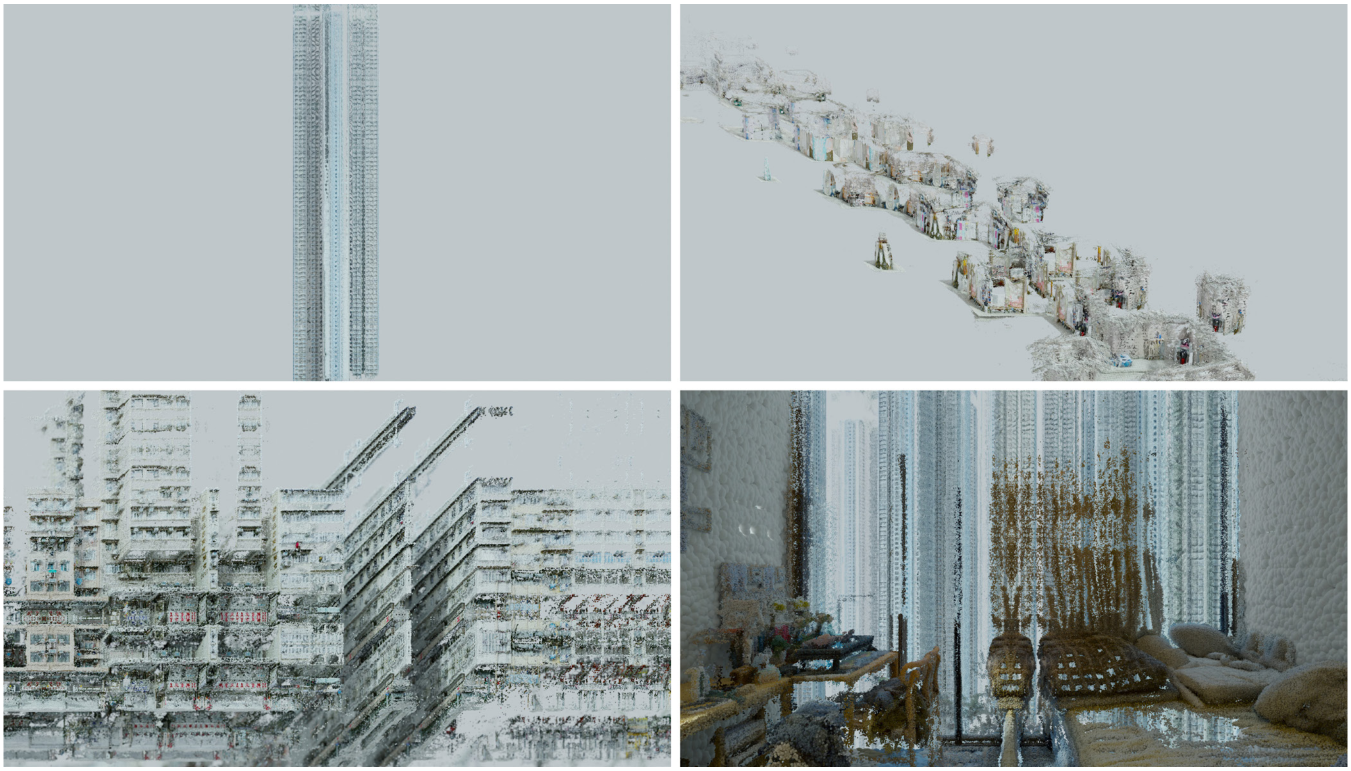
RESULTS AND REFLECTION

As the direct outcomes of the research, the two animated films, 4'42" and 3'13" in length, demonstrated the capabilities and efficiency of the representational workflow using photogrammetric data and Blender geometry node. Among the limited use cases of the ply-import function since its introduction in 2022, this project has pushed the boundaries of its performance and expression, with a high level of completeness and complexity, as well as a specific focus on architectural space and form.

The films present a visual expression of point cloud that is less driven by errors or constraints, but more by its ability to indicate spatial transformation, focusing on its malleability as a digital *material,* rather than the biases in information transmission as a *medium*. The project refrained from designing forms, maintaining the scanned models in their original state to focus on the folding of space. This choice aims to intensify the expression of spatial operations while avoiding fixation on content. Nonetheless, similar to other visual works using point cloud, the realistic quality of scanned data contributes to the aesthetic tension between the digital and the real. Contrary to the prevalent desire to break away from reality in imagining the future digital space, the films offer an alternative that faithfully archives past experiences, therefore preserving the memories of a rapidly changing city.

Through filmic experiences, the project suggests new possibilities in virtual space design. By utilizing point clouds to showcase the transformation of space, it presents a viable approach for designers to engage in non-3D spatial experiments, which is to construct a transformable object through procedural modeling and render its current 3D section or projection. Compared to building a non-3D engine, this shortcut enables fast design iterations to test spatial experiences and aesthetic affects. It also identifies design opportunities unique to the digital realm: the spatial topology and dimensionality could be the subject of design. This leads to new experiences and forms, as the two films demonstrate respectively: *Forking Rooms* depicts an uncanny journey through teleconnected rooms and reciprocal transformations between a room and a picture. *Fractal Cities* visualizes how found spaces can be morphed, folded, and multiplied to generate an unseen representation of the city. Both films aim to offer tangible insights into the nature of non-3D spatial experiences and their distinctiveness.

The experiments interrogate fundamental architectural concepts in the digital realm, *boundary*, *scale*, and *directionality*, paving the way for new foundations in virtual space creation. Each prototype resonates with a specific mathematical definition and displays experiential and aesthetic characteristics. Together, they offer preliminary examples in the search for a richer lexicon.



12 Stills from *Fractal Cities*.

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In Western painting, *mise en abyme* is a technique to construct self-reference by transposing a part of the space onto itself. Victor Stoichita writes in the discussion of *Las Meninas*, a classic demonstration of the technique, "*Las Meninas* is an 'open' painting, not only because it is 'infinitely interpretable,' but also from the point of view of its spatial structure" (Stoichita 1997). The painting generates aporia and positions art as a problem. Such spatial insolubility is erased in a typical 3D scene, where objects with fixed locations and confined shapes compose a world of singular truth. As a critique, the project aims to retrieve the room for paradox and fluidity in virtual environments, recalling the freedom that the digital space once promised.

Limitations and Future Goals

The current outcome only accounts for non-3D filmic experience, but not yet non-3D space. Although the project could be upgraded to a first-person VR environment, with foreseeable hardware advancement, the challenge lies in the inclusion of multiple spectators. As some of the current transformations cater to a single vantage point, it may take a universal function to resolve the spatial transformation that involves various user positions.

Greater mathematical rigor can also frame the study more systematically, and guide the future quest for spatial prototypes. The project should continue to seek mathematical tools capable of describing and constructing non-3D spaces, as well as efficient methods for translating them into code. Also, the prototypes only cover limited non-Euclidean, fractal, or hyperdimensional instances, each of which is a vast family to be investigated.

CONCLUSION

The study expands the capacity of point cloud in architecture and film, representing with it the transformation of found cityscapes and interior scenes. Point cloud operations serve as design vocabularies for storytelling and space-making, offering novel spatial experiences and building forms that enrich the current understanding of the medium's distinct qualities. Method-wise, the project manipulates point cloud with the Blender geometry node, which improves the efficiency of programming the point cloud and rendering quality compared to previous tools. It manifests the capacity with precise geometric transformations categorized as three spatial prototypes.

The project also substantiates the experience of non-3D space using film. The spatial prototypes draw on non-Euclidean, fractal, and hyperdimensional - mathematical concepts that challenge flat, 3D spaces. They are constructed through procedural modeling as transformable models morphing in response to the vantage point or time, whose experience is visualized through film. The project proposes point cloud as a suitable medium to represent the experience of non-3D spaces, and outlines a feasible path for future experiments. It not only suggests new design opportunities in virtual environments, but also anticipates a future digital world with fluid spatial structures.

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IMAGE CREDITS

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